Note that we can equivalently solve games of incomplete information from the perspective of an agent who knows their type, or from the perspective of an agent who doesn’t yet know their type and chooses a strategy mapping from the set of all possible types to actions. In this problem we do the latter, but in, problem 2 we do the former. These two methods are equivalent, so you should use whichever is easier for you in the given problem.

2.1

Solution

Firm profits are:
\[ \Pi_1 = q_1 (1 - q_{21} - q_1)/3 + q_1 (1 - q_{22} - q_1)/3 + q_1 (1 - q_{23} - q_1)/3 \]
\[ \Pi_{21} = q_{21} (1/2 - q_{21} - q_1) \]
\[ \Pi_{22} = q_{22} (1 - q_{22} - q_1) \]
\[ \Pi_{23} = q_{23} (3/2 - q_{23} - q_1) \]

Best responses are:
\[ q_1 = (1 - q_{21}/3 - q_{22}/3 - q_{23}/3)/2 \]
\[ q_{21} = (1/2 - q_1)/2 \]
\[ q_{22} = (1 - q_1)/2 \]
\[ q_{23} = (3/2 - q_1)/2. \]

Then, substituting in, we have
\[ q_1 = (1 - (1/2 - q_1)/2/3 - (1 - q_1)/2/3 - (3/2 - q_1)/2/3)/2 = 1/4 - q_1/4 \]
\[ q_1 = \frac{1}{3} \]
\[ q_{21} = \frac{1}{12} \]
\[ q_{22} = \frac{1}{3} \]
\[ q_{23} = \frac{7}{12}. \]

2.2

Solution

Firm profits are:

\[ \Pi_{11} = q_{11}(1/2 - q_{21} - q_{11})/2 + q_{11}(1/2 - q_{22} - q_{11})/2 = q_{11}(1/2 - q_{21}/2 - q_{22}/2 - q_{11}) \]
\[ \Pi_{12} = q_{12}(3/2 - q_{21} - q_{12})/2 + q_{12}(3/2 - q_{22} - q_{12})/2 = q_{12}(3/2 - q_{21}/2 - q_{22}/2 - q_{12}) \]
\[ \Pi_{21} = q_{21}(1/2 - q_{11}/2 - q_{12}/2 - q_{21}) \]
\[ \Pi_{22} = q_{22}(3/2 - q_{11}/2 - q_{12}/2 - q_{22}) \]

Best responses are:

\[ q_{11} = 1/4 - q_{21}/4 - q_{22}/4 \]
\[ q_{12} = 3/4 - q_{21}/4 - q_{22}/4 \]
\[ q_{21} = 1/4 - q_{11}/4 - q_{12}/4 \]
\[ q_{22} = 3/4 - q_{11}/4 - q_{12}/4. \]

Then, substituting in, we have

\[ q_{11} = 1/4 - (1/4 - q_{11}/4 - q_{12}/4)/4 - (3/4 - q_{11}/4 - q_{12}/4)/4 = (q_{11} + q_{12})/8 \]
\[ q_{12} = 3/4 - (1/4 - q_{11}/4 - q_{12}/4)/4 - (3/4 - q_{11}/4 - q_{12}/4)/4 = 1/2 + (q_{11} + q_{12})/8 \]
\[ q_{21} = (q_{21} + q_{22})/8 \]
\[ q_{22} = 1/2 + (q_{21} + q_{22})/8 \]

We now have systems of two equations in two unknowns, which we can solve easily:

\[ q_{11} = (q_{11} + q_{12})/8 \]
\[ 7q_{11} = q_{12} \]
\[ q_{12} = 1/2 + (q_{11} + q_{12})/8 \]
\[ 7q_{11} = 1/2 + (q_{11} + 7q_{11})/8 \]
\[ 7q_{11} = 1/2 + q_{11} \]
\[ q_{11} = 1/12 \]
\[ q_{12} = 7/12 \]
\[ q_{21} = 1/12 \]
\[ q_{22} = 7/12 \]
3
Solution
http://home.uchicago.edu/~hanzhe/teaching/ECON207/207lec_auction.pdf
Section 3

4
Solution
Problem 2 in old solutions

5
Solution
In notes: http://www.unc.edu/~normanp/711part7.pdf

6
Solution
1. This is a mechanism design problem, where we want to construct a mechanism (we’re allowed to choose any q we want) such that agents have no incentive to misreport their true type. In principle, we have 4 inequalities we must satisfy. Define the expected payoff of player i reporting type \( \hat{\theta} \) when their true type is \( \theta \) as \( \Pi_i(\hat{\theta}, \theta) \). Note that the payoffs for player 1 are the settlement values, minus court fees if they go to court, and for player 2 the payoffs are the settlement values times -1, minus court fees if they go to court. It must be the case that:

\[
\Pi_1(w, w) \geq \Pi_1(s, w)
\]

That is, if player 1 is weak, they are weakly better off reporting weak than strong

\[
\Pi_1(s, s) \geq \Pi_1(w, s)
\]

That is, if player 1 is strong, they are weakly better off reporting strong than weak

\[
\Pi_2(w, w) \geq \Pi_2(s, w)
\]

That is, if player 2 is weak, they are weakly better off reporting weak than strong

\[
\Pi_2(s, s) \geq \Pi_2(w, s)
\]
That is, if player 2 is strong, they are weakly better off reporting strong than weak.

We can ignore inequalities 2 and 4, though, as strong types never have an incentive to misreport. Misreporting your type as weak has two effects: 1) your payoff in an out of court settlement is decreased by 8,000 2) your likelihood of settling out of court increases. Even taking into account the lack of court fees with an out of court settlement, your payoff will be 6,500 lower settling out of court with a report of weak than going to court. Thus, the payoff for reporting weak will always be lower, since settling out of court is always worse for you.

Let’s now consider the choice of report when your true type is weak:

\[ \Pi_1(w, w) \geq \Pi_1(s, w) \]

\[ \frac{1}{2}(q2,000+(1-q)(2,000-1,500)) + \frac{1}{2}(10,000) \geq \frac{1}{2}(2,000-1,500) + \frac{1}{2}(q18,000+(1-q)10,000) \]

\[ 0 \geq q \]

\[ \Pi_2(w, w) \geq \Pi_2(s, w) \]

\[ \frac{1}{2}(q(-10,000-1,500)+(1-q)(-10,000)) \geq \frac{1}{2}(-18,000-1,500) + \frac{1}{2}(q(-2,000)+(1-q)(-10,000-1,500)) \]

\[ 1 \geq q \]

Thus it must be that \( q = 0 \).

2. Given \( q = 0 \), if you are player 1 and your type is strong, you know you will go to court. Thus your expected payoff is \( \frac{1}{2}(10,000) + \frac{1}{2}(18,000) - 1,500 = 12,500 \). If Player 2 is strong, his expected payoff for using the mechanism is \( \frac{1}{2}(-10,000) + \frac{1}{2}(-2,000) - 1,500 = -7,500 \). If Player 2 is weak, his expected payoff is \( \frac{1}{2}(-10,000) + \frac{1}{2}(-18,000 - 1,500) = -14,750 \). Thus, player 2 will only accept this offer if he is weak, in which case player 1 would be better off staying with the mechanism, going to court, and getting 18,000 - 1,500 = 16,500. Thus player 1 should not listen to their lawyer.