Assignment 3

1. Consider the static game

\[
\begin{array}{cc}
L & R \\
U & a_1, a_2 \quad b_1, b_2 \\
D & c_1, c_2 \quad d_1, d_2 \\
\end{array}
\]

Using this static game, construct a sequential game by adding messages from player 1 to player 2. That is, in stage one player 1 picks a message \(m \in \{u, d\}\) that is observed by player 2. Then, in stage 2, they play the static game above and the payoffs depend only on the outcome in the second stage and not the message \(m\).

1. Consider the case with \(a_1 = d_2 = 2\), \(a_2 = d_1 = 1\) and \(b_1 = b_2 = c_1 = c_2 = 0\). Is there a subgame perfect equilibrium where the actions played after \(u\) are different from the action played after \(d\)? Which actions will be played in the second stage in such an equilibrium.

2. Consider the case with \(a_1 = a_2 = 1\), \(d_1 = d_2 = 2\), \(b_1 = c_2 = 3\), and \(c_1 = b_2 = 0\). Is there a subgame perfect equilibrium where the actions played after \(u\) are different from the action played after \(d\)?

2. Consider the coordination game,

\[
\begin{array}{cc}
A & B \\
A & 10, 10 \quad 0, 0 \\
B & 0, 0 \quad 1, 1 \\
\end{array}
\]

1. Find all pure strategy Nash equilibria.

2. Now, amend the game as follows. Let the strategy set for player \(i\) be \(\{A, B\} \times N\) where \(N\) denotes the set of integers. Also, assume that

\[
U_i (A, A, k_i, k_j) = \begin{cases} 
9 & \text{if } k_i < k_j \\
10 & \text{if } k_i = k_j \\
11 & \text{if } k_i > k_j \\
\end{cases}
\]

\[
U_i (A, B, k_i, k_j) = 0 \text{ for all } k_i, k_j
\]

\[
U_i (B, A, k_i, k_j) = 0 \text{ for all } k_i, k_j
\]

\[
U_i (B, B, k_i, k_j) = 1 \text{ for all } k_i, k_j
\]

Find all Nash equilibria.