1. Consider a prisoners dilemma with stage game payoffs

\[
\begin{array}{cc}
c & d \\
C & 2, 2 & 0, 3 \\
D & 3, 0 & 1, 1 \\
\end{array}
\]

played twice.

1. Sketch the extensive form.
2. How many nodes are there in the extensive form? How many information sets? How many pure strategies?
3. Write down the payoff matrix for the reduced normal form. Assume that there is no discounting.
4. Find all pure strategy Nash equilibria of the reduced normal form game.
5. What are all Nash equilibria for the regular normal form? For each equilibrium, specify the equilibrium outcome and determine which equilibria are subgame perfect.
7. Now consider an arbitrary finite horizon \( T \). Prove that no player can play \( C \) on the equilibrium path.
8. Given an arbitrary finite horizon \( T \), write down a strategy profile that is a Nash equilibrium, but not subgame perfect.

2. Consider the normal form game

\[
\begin{array}{ccc}
c & d_A & d_B \\
C & 3, 3 & -2, 4 & -2, 4 \\
D_A & 4, -2 & 2, 2 & -1, -1 \\
D_B & 4, -2 & -1, -1 & 0, 0 \\
\end{array}
\]

1. Find all Nash equilibria (pure and mixed).
2. Suppose that the stage game is played twice and that payoffs are not discounted. Construct a subgame perfect equilibrium in which \((C, c)\) is played in the first period?
3. Suppose that the stage game is repeated 4 times. Construct a subgame perfect equilibrium where \((D_A, c)\) is played in the first period?
4. Consider the following outcome path \((D_A, d_B), (D_B, d_A), (D_A, d_A), ..., (D_A, d_A)\). Construct a strategy profile that supports this as a subgame perfect equilibrium in the \( T \) repeated game. What is the smallest value of \( T \) for this to work?

3. Consider an infinite repetition of

\[
\begin{array}{cc}
c & d \\
C & 2, 2 & 0, 3 \\
D & 3, 0 & 1, 1 \\
\end{array}
\]

and assume payoffs are discounted by \( \delta \in (0, 1) \).

1. For \( \delta \) large enough, construct a subgame perfect equilibrium where \((C, c)\) is played in every period on the equilibrium path. Use Nash reversion in your construction and find a critical value for \( \delta \).
2. Change the stage game to

<table>
<thead>
<tr>
<th></th>
<th>p</th>
<th>c</th>
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<tbody>
<tr>
<td>P</td>
<td>-1, -1</td>
<td>-1, -1</td>
<td>-1, 0</td>
</tr>
<tr>
<td>C</td>
<td>-1, -1</td>
<td>2, 2</td>
<td>0, 3</td>
</tr>
<tr>
<td>D</td>
<td>0, -1</td>
<td>3, 0</td>
<td>1, 1</td>
</tr>
</tbody>
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Assuming strategies relying on Nash reversion, under which condition on $\delta$ can a subgame perfect equilibrium where $(C, c)$ is played in every period on the equilibrium path be supported.

3. Given a normal form game $(N, S, u)$, let $u_i = \min_{s\in S_i} \max_{s'\in S_i} u_i(s_i, s_{-i})$ be the (pure strategy) minmax value for player $i$. What are the minmax values in the two games above.

4. Construct a strategy with the following properties: 1) $(C, c)$ is played in every period on the outcome path; 2) if $(D, c)$ is played when $(C, c)$ should have been played $(P, d)$ is played in the next period; if $(C, d)$ is played when $(C, c)$ would have been played $(D, p)$ is played in the next period. For any any history where $(P, d)$ or $(D, p)$ has been played at least 1 plays $D$ and 2 plays $d$. Write down a fully specified strategy for each player so that these properties hold and find a critical value for $\delta$ so that the strategy profile is subgame perfect.

4. Consider a Cournot game with inverse demand $p(y) = 12 - y$ and constant marginal cost $c = 3$.

1. Find the static Nash equilibrium in the one shot Cournot game and find the “cartel outcome” that maximizes industry profits.
2. Suppose the game is repeated $T < \infty$ times. Find all subgame perfect equilibria.
3. Suppose that payoffs are discounted by $\delta$ and that the game is repeated infinitely. Derive a condition under which it is possible to sustain industry profits at the level of the cartel outcome in every period as a subgame perfect equilibrium using Nash reversion strategies.
4. Suppose that your condition on $\delta$ fails, so that the cartel outcome cannot be supported by Nash reversion. Can you sustain any other outcome than the stage game Nash if you restrict attention to Nash reversion strategies.

5. Let $G$ be an arbitrary stage game and let $G^T$ be the $T$-fold repetition. A strategy profile is history independent if for every $t$ the continuation strategies satisfy $s|h_t = s|h'_t$ for every $h_t, h'_t \in H_t$.

1. True/False? Any history independent strategy that is subgame perfect must induce stage game Nash equilibrium play in every period. (provide a proof or counter example)
2. True/False? Any history independent strategy profile that induces Nash equilibrium play in every period is subgame perfect. (proof or counter example)
3. True/False? Every strategy profile that induces stage game Nash equilibrium play in every subgame is subgame perfect. (proof or counter example).